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# A Heuristic-Based Decision Tree for Connected Components Labeling of 3D Volumes



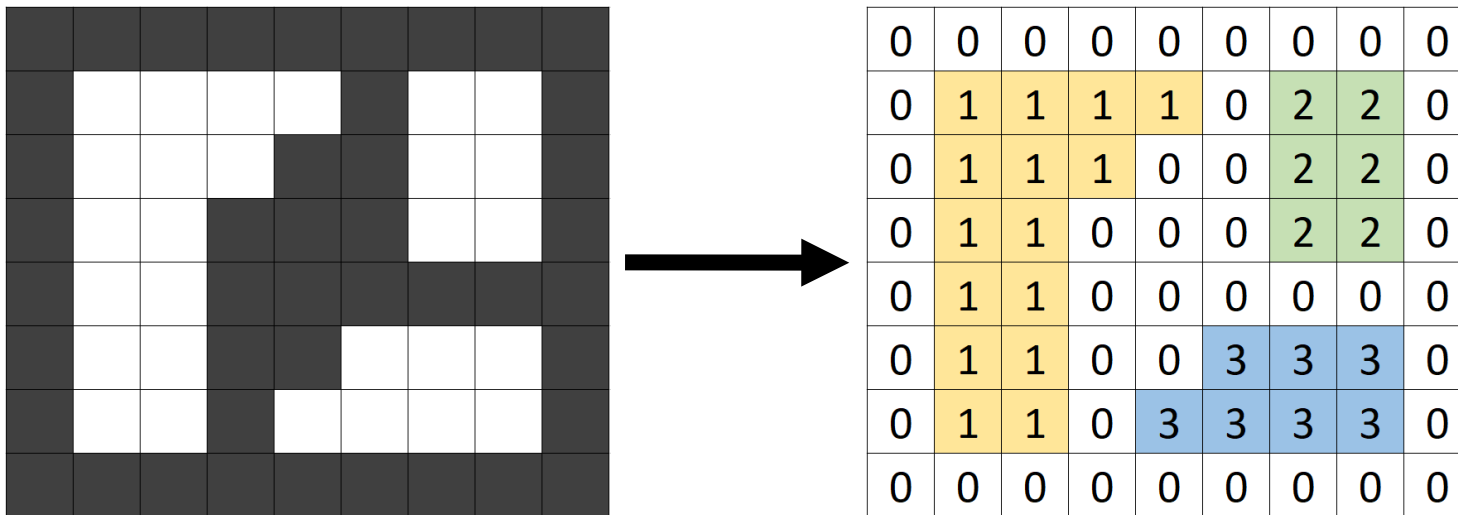
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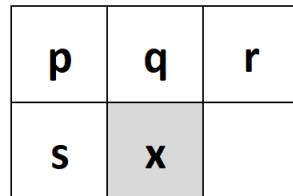
# Connected Components Labeling (CCL)

- Find all connected, foreground pixel regions within a binary image
- Each pixel region, or **connected component**, receives a unique label
- Fundamental for image segmentation and object recognition
- CCL should be as fast as possible

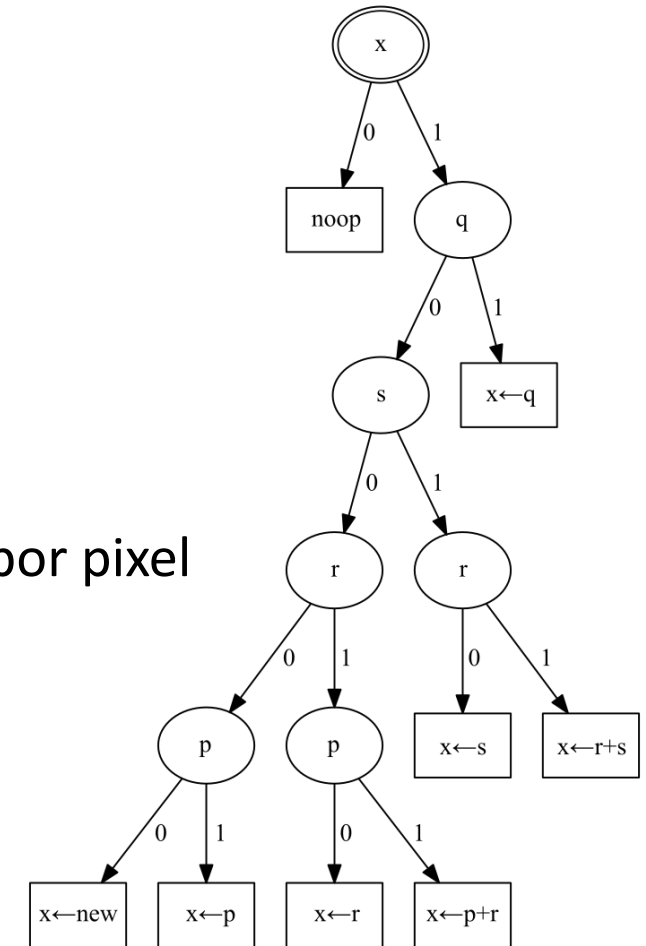


# History of CCL Research (1/2)

- Rosenfeld and Pfaltz<sup>1</sup> invented **two scans** algorithms
  - First scan: gives each pixel a **provisional label** based on the neighborhood mask, and **solves label equivalences**
  - Second scan: assigns **definitive labels**
- Wu et al.<sup>2</sup> proposed **Optimal Decision Trees (ODTs)**
  - Label for a pixel can be decided **without reading** every neighbor pixel
  - Optimal binary decision tree minimizes pixel reads
  - Each tree node represents a pixel read



Rosenfeld mask



Optimal binary decision tree (ODT)  
for the Rosenfeld mask

<sup>1</sup>A. Rosenfeld and J. L. Pfaltz, "Sequential Operations in Digital Picture Processing," *Journal of the ACM*, vol. 13, no. 4, pp. 471–494, Oct. 1966.

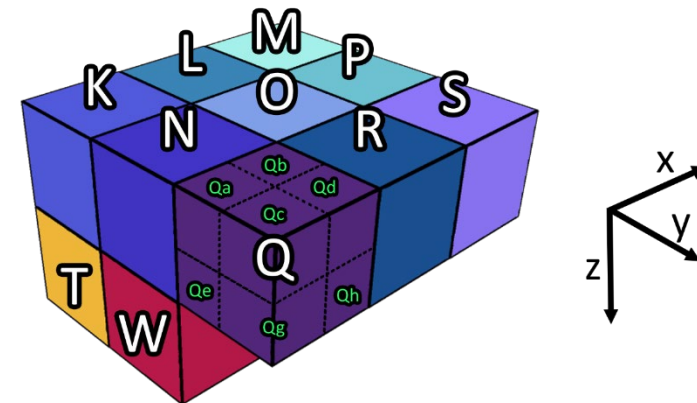
<sup>2</sup>K. Wu, E. Otoo, and K. Suzuki, "Two Strategies to Speed up Connected Component Labeling Algorithms," Lawrence Berkeley National Laboratory, Tech. Rep. LBNL-59102, 2005.

# History of CCL Research (2/2)

- Grana et al.<sup>1</sup> proposed **block-based scan mask**
  - In a 2x2 block, all foreground pixels share the same label
  - Generated the ODT **automatically** since it is unfeasible by hand:  $2^{16}$  cases, 136 nodes
- **What about 3D CCL?**
  - Multiple possible block-based masks: 2x1x1, 2x2x1 and 2x2x2
  - Explosion in computational complexity makes the ODT generation **infeasible**
  - **No existing 3D CCL algorithm employs a block-based mask**
  - Goal: generate a **near-optimal tree** with a heuristic strategy

(a)	b	c	d	e	(f)
g	h	i	j	k	(l)
m	n	o	p		
(q)	r	s	t		

2x2 Grana mask



2x2x2 voxel mask

<sup>1</sup>C. Grana, D. Borghesani, and R. Cucchiara, "Optimized Block-based Connected Components Labeling with Decision Trees," *IEEE Transactions on Image Processing*, vol. 19, no. 6, pp. 1596–1609, 2010.

# Heuristics – Concept

- **Shannon Entropy** (information theory)

- Given a set of events  $E$ , with  $P_i$  being the probability of an event  $i \in E$ , the entropy  $H_E$  is:

$$H_E = \sum_i -P_i \log P_i$$

- Entropy describes the uncertainty of outcomes

- **Decision Tree Learning**

- Generate decision trees for complex datasets quickly
- **Recursively partition** the dataset through entropy calculation
  1. Try *splitting* on every attribute
  2. Calculate **Information Gain (IG)** on subsets – IG measures average entropy reduction
  3. Apply *split* with the highest information gain
- For CCL, the dataset is the decision table

							assign				merge	
x	p	q	r	s	no action	new label	x = p	x = q	x = r	x = s	x = p+r	x = r+s
0	-	-	-	-	1							
1	0	0	0	0		1						
1	1	0	0	0			1					
1	0	1	0	0				1				
1	0	0	1	0					1			
1	0	0	0	1						1		
1	1	1	0	0			1	1				
1	1	0	1	0							1	
1	1	0	0	1			1			1		
1	0	1	1	0				1	1			
1	0	1	0	1				1		1		
1	0	0	1	1								1
1	1	1	1	0			1	1	1			
1	1	1	0	1			1	1		1		
1	1	0	1	1							1	1
1	0	1	1	1				1	1	1		
1	1	1	1	1			1	1	1	1		

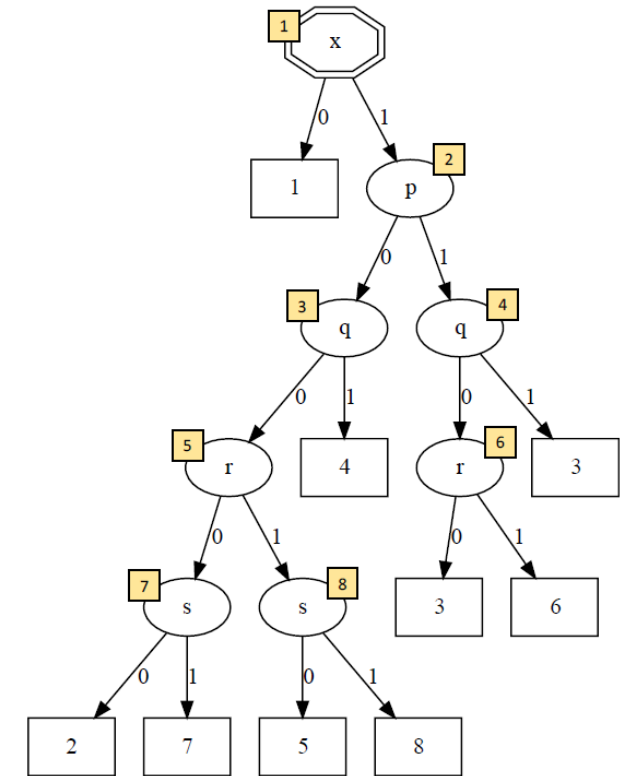
OR-decision table for the Rosenfeld mask

# Applying Decision Tree Learning to CCL

- **Entropy Partitioning Decision Tree (EPDT)** generated for the Rosenfeld mask
- For each node, the pixel with maximum IG is chosen

Node	Depth	$H(S)$	p			q			r			s			x		
			$H_0$	$H_1$	IG	$H_0$	$H_1$	IG	$H_0$	$H_1$	IG	$H_0$	$H_1$	IG	$H_0$	$H_1$	IG
1	0	2.2	2.0	1.4	0.5	2.3	1.5	0.3	1.9	2.1	0.2	2.1	2.1	0.1	<b>0.0</b>	<b>2.4</b>	<b>1.0</b>
2	1	2.4	<b>2.0</b>	<b>0.8</b>	<b>1.0</b>	2.5	1.0	0.7	1.8	2.3	0.4	2.2	2.2	0.2			
3	2	2.0				<b>2.0</b>	<b>0.0</b>	<b>1.0</b>	1.5	1.5	0.5	1.5	1.5	0.5			
4	2	0.8				<b>1.0</b>	<b>0.0</b>	<b>0.3</b>	0.0	1.0	0.3	0.8	0.8	0.0			
5	3	2.0							<b>1.0</b>	<b>1.0</b>	<b>1.0</b>	1.0	1.0	1.0			
6	3	1.0							<b>0.0</b>	<b>0.0</b>	<b>1.0</b>	1.0	1.0	0.0			
7	4	1.0										<b>0.0</b>	<b>0.0</b>	<b>1.0</b>			
8	4	1.0										<b>0.0</b>	<b>0.0</b>	<b>1.0</b>			

- The result is **near-optimal**
  - Only one more node than the optimal decision tree
- Next step: apply EPDT to 3D block-based masks



EPDT for the Rosenfeld mask

# 3D EPDT algorithms

- New 3D EPDT CCL algorithms
- Varying block size and number of pixels
- **EPDT\_19c**
  - Block size 2x1x1
  - Smallest 3D block-based mask
- **EPDT\_22c**
  - Block size 2x1x1
  - Add borders pixels, for more efficient actions
- **EPDT\_26c**
  - Block size 2x2x1
  - Largest tree that compilers can handle

Ka	Kb	La	Lb	Ma	Mb
Na	Nb	Oa	Ob	Pa	Pb
Qa	Qb	Ra	Rb	Sa	Sb

Ta	Tb	Ua	Ub	Va	Vb
Wa	Wb	Xa	Xb		

EPDT\_19c 3D mask

Ka	Kb	La	Lb	Ma	Mb
Na	Nb	Oa	Ob	Pa	Pb
Qa	Qb	Ra	Rb	Sa	Sb

Ta	Tb	Ua	Ub	Va	Vb
Wa	Wb	Xa	Xb		

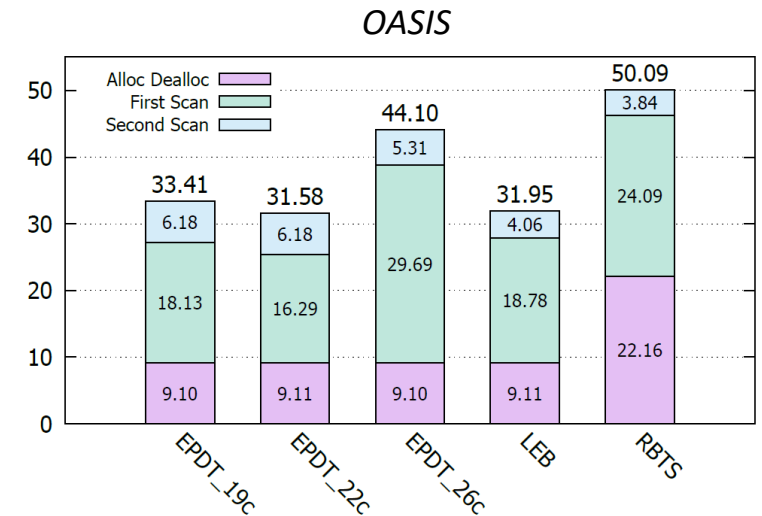
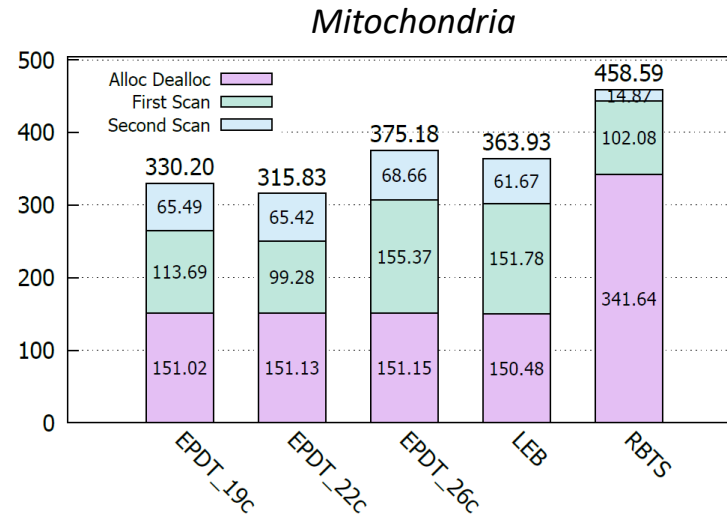
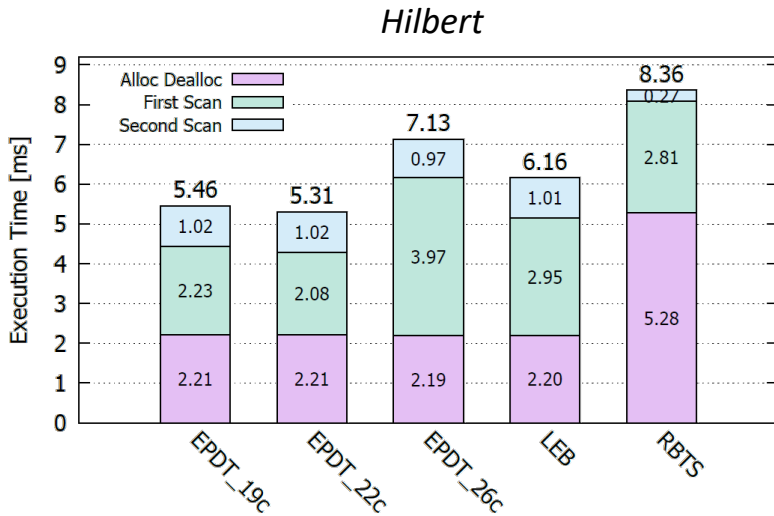
EPDT\_22c 3D mask

Ka	Kb	La	Lb	Ma	Mb
Kc	Kd	Lc	Ld	Mc	Md
Na	Nb	Oa	Ob	Pa	Pb
Nc	Nd	Oc	Od	Pc	Pd
Qa	Qb	Ra	Rb	Pa	Pb
Qc	Qd	Rc	Rd	Pc	Pd

Ta	Tb	Ua	Ub	Va	Vb
Tc	Td	Uc	Ud	Vc	Vd
Wa	Wb	Xa	Xb		
Wc	Wd	Xc	Xd		

EPDT\_26c 3D mask

# Experimental Results



- EPDT algorithms improve the performance of the first scan by saving many **memory accesses**
- EPDT\_26c has a very large decision tree → bad impact on instruction cache
- EPDT\_22c improves current state-of-the-art<sup>1</sup>

Algorithm	Binary Image	Labels Image	Equivalences Vector	Total
LEB	11.461	27.182	9.851	48.494
EPDT_19c	14.917	17.760	1.169	33.846
EPDT_22c	14.057	17.753	1.145	32.955
EPDT_26c	13.695	13.145	0.728	27.568

Average number of load/store operations on the OASIS dataset, expressed in millions.

<sup>1</sup>L. He, Y. Chao, and K. Suzuki, "Two Efficient Label-Equivalence-Based Connected-Component Labeling Algorithms for 3-D Binary Images," *IEEE Transactions on Image Processing*, vol. 20, no. 8, pp. 2122–2134, 2011.